Kepler's Laws and Gravity

Physics
Unit 05

NAD 2023 Standards
- Gravity G1
- Gravity G2

Credits

- This Slideshow was developed to accompany the textbook
  - *OpenStax High School Physics*
    - *Available for free at https://openstax.org/details/books/physics*
    - *By Paul Peter Urone and Roger Hinrichs*
    - *2020 edition*
- Some examples and diagrams are taken from the *OpenStax College Physics, Physics,* and *Cutnell & Johnson Physics* 6th ed.

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5-01 Kepler's Laws of Planetary Motion

In this lesson you will…

- Predict the motion of orbiting objects in the solar system
- Explain Kepler's Laws of Planetary Motion
- Diagram a planets elliptical orbit

NAD 2023 Standards
Gravity G2

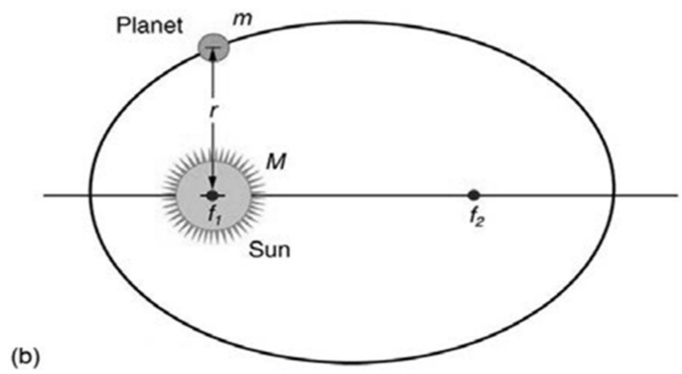
OpenStax High School Physics 7.1
OpenStax College Physics 2e 6.6

5-01 Kepler's Laws of Planetary Motion

- After studying motion of planets, Kepler came up with his laws of planetary motion
- Newton then proved them all using his Universal Law of Gravitation
- Assumptions:
  - A small mass, m, orbits much larger mass, M, so we can use M as an approximate inertia reference frame
  - The system is isolated

5-01 Kepler's Laws of Planetary Motion

1. The orbit of each planet about the Sun is an ellipse with the sun at one focus.



- Closest point to sun perihelion

- Farthest distance to sun aphelion

Watch video Kepler's First Law

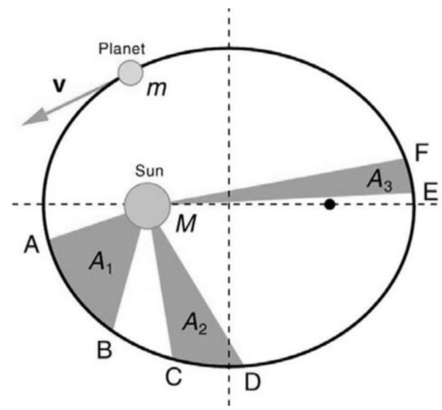
5-01 Kepler's Laws of Planetary Motion

- Do Kepler's First Law lab

5-01 Kepler's Laws of Planetary Motion

2. Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times.

Watch video Kepler's Second Law



5-01 Kepler's Laws of Planetary Motion

3. The ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun.

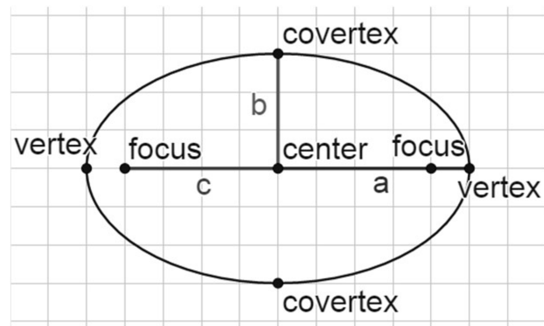
$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

- Watch Kepler's Third Law video

T is period and r is average orbital radius

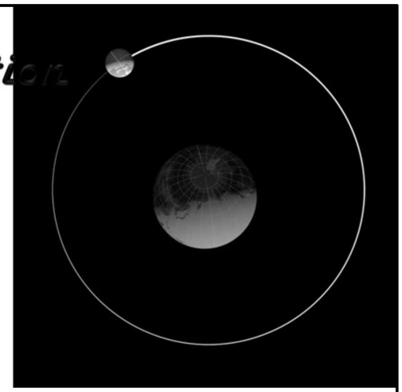
5-01 Kepler's Laws of Planetary Motion

- Parts of an ellipse
  - a = semimajor axis (distance from center to vertex)
    - $a = \frac{r_a + r_p}{2}$
  - b = semiminor axis (distance from center to covertex)
    - $b = \sqrt{r_a r_b}$
  - c = focal length (distance from center to focus)
    - $c = r_a - a$
- Area of ellipse
  - $A = \pi ab$



- Eccentricity
  - Measure of how stretched out an ellipse is
  - $e = \frac{c}{a}$
  - e = 0 circle, e = 1 line

5-01 Kepler's Laws of Planetary Motion



- The perihelion of the moon from earth is 358000 km. Its aphelion is 399000 km. What is the moon's orbit's semimajor axis, semiminor axis, focal length, and eccentricity?

$$a = \frac{r_a + r_b}{2} = \frac{3.58 \times 10^8\ m + 3.99 \times 10^8\ m}{2} = 3.785 \times 10^8\ m$$

$$b = \sqrt{r_a r_b} = \sqrt{(3.58 \times 10^8\ m)(3.99 \times 10^8\ m)} = 3.779 \times 10^8\ m$$

$$r_a = a + c$$

$$3.99 \times 10^8\ m = 3.785 \times 10^8\ m + c$$

$$c = 0.205 \times 10^8\ m = 2.05 \times 10^7\ m$$

$$e = \frac{c}{a} = \frac{2.05 \times 10^7\ m}{3.785 \times 10^8\ m} = 0.054$$

5-01 Kepler's Laws of Planetary Motion

- If it takes 27.3 days for the moon to orbit the earth, how much area does a line from the earth to the moon sweep out every day?

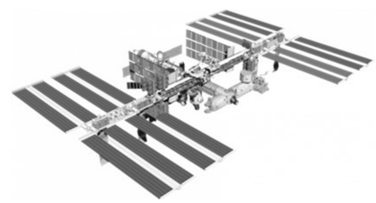
$$\frac{Area}{time}$$

$$\frac{\pi ab}{t}$$

$$\frac{\pi(3.785 \times 10^{8}\ m)(3.779 \times 10^{8}\ m)}{27.3\ d} = 1.65 \times 10^{16}\,\frac{m^2}{d}$$

5-01 Kepler's Laws of Planetary Motion

- The moon's average radius of orbit is 384,399 km and takes 27.322 days to orbit the earth. The International Space Station's average radius of orbit is 417.5 km above the earth. What is the period of the ISS's orbit?



$$r_{ISS} = 417.5\ km + 6380\ km = 6797.5\ km$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\frac{T_1^2}{(27.322\ d)^2} = \frac{(6797.5\ km)^3}{(384399\ km)^3}$$

$$T_1^2 = \frac{(6797.5\ km)^3}{(384399\ km)^3}(27.322\ d)^2 = 0.0041279\ d^2$$

$$T_1 = 0.0642\ d = 1.54\ h$$

5-02 Weight and Gravity

In this lesson you will…

- Use Newton's Law of Gravitation to describe the gravitational forces between objects
- Find the acceleration due to gravity for various locations

NAD 2023 Standards
Gravity G2

OpenStax High School Physics 7.2
OpenStax College Physics 2e 6.5

5-02 Weight and Gravity

- Every particle in the universe exerts a force on every other particle

$$F_g = G\frac{mM}{r^2}$$

- $G = 6.673 \times 10^{-1} \quad N\, m^2/kg^2$
- m and M are the masses of the particles
- r = distance between the particles (centers of objects)

G is the universal gravitational constant – measured by Henry Cavendish using a very sensitive balance 100 years after Newton proposed the law

5-02 Weight and Gravity

- For bodies
- Using calculus – apply universal gravitation for bodies

- Estimate (quite precisely)
  - Assume bodies are particles based at their center of mass
  - For spheres assume they are particles located at the center

5-02 Weight and Gravity

- What is the gravitational attraction between a 75-kg boy (165 lbs) and the 50-kg girl (110 lbs) seated 1 m away in the next desk?

- $F_g = 2.5 \times 10^{-7}$ N
  - $= 2.6 \times 10^{-8}$ lbs of force

$$m_1 = 75\ kg;\ m_2 = 50\ kg;\ r = 1\ m$$

$$F_g = \frac{GMm}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \frac{Nm^2}{kg^2}\left(75\ kg\right)\left(50\ kg\right)}{\left(1\ m\right)^2}$$

$$= 2.5 \times 10^{-7}\ N$$

5-02 Weight and Gravity

- Weight is Gravitational Force the earth exerts on an object
- Unit: Newton (N)
- Remember!!!
  - Weight is a Force

Eureka #7

5-02 Weight and Gravity

- Since weight is the force of gravity

$$W = G\frac{mM}{r^2}$$
$$W = mg$$

$$g = G\frac{M}{r^2}$$

- r is usually R_E
    - So g = 9.80 m/s^2

5-02 Weight and Gravity

- Find the acceleration due to gravity at the altitude of the ISS, 417.5 km above the earth.

- $g = 8.64\ m/s^2$ not much smaller than $9.8\ m/s^2$

$$r_{ISS} = 417.5 \times 10^3\ m + 6.38 \times 10^6\ m = 6.7975 \times 10^6\ m$$

$$g = \frac{GM}{r^2}$$

$$g = \frac{\left(6.673 \times 10^{-11}\,\frac{Nm^2}{kg^2}\right)(5.98 \times 10^{24}\ kg)}{(6.7975 \times 10^6\ m)^2} = 8.64\ m/s^2$$

5-02 Weight and Gravity

- The gravitational pull from the moon and sun causes tides
  - Water is pulled in the direction of the moon and sun

- Gravitational pull from satellites causes the main body to move slightly
  - Moon causes earth to move
  - Planets cause sun/star to move

5-03 Satellites

In this lesson you will…

- Use Newton's 2nd law of motion, Newton's law of gravitation, and centripetal force to find the speed of the orbit of a satellite
- Determine the period of an object in orbit

NAD 2023 Standards
Gravity G2

Not in OpenStax High School Physics
OpenStax College Physics 2e 6.6

5-03 Satellites

- Satellites
  - Any object orbiting another object only under the influence of gravity
  - One way to find the speed of a satellite in a circular orbit
  - $v = \frac{d}{t}$
  - $v = \frac{2\pi r}{T}$
    - Where r = orbital radius, T = period of orbit

5-03 Satellites

- Gravity provides the centripetal force
- There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.
- Why only one speed?
- $F_c = \frac{mv^2}{r}$ $\qquad F_g = \frac{GMm}{r^2}$
- $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$v = \sqrt{\frac{GM}{r}}$$

  - *r* is measured from the center of the earth

5-03 Satellites

$$v = \sqrt{\frac{GM}{r}}$$

- Since $1/r$
  - As r decreases, v increases
- Mass of the satellite is not in the equation, so speed of a massive satellite = the speed of a tiny satellite

5-03 Satellites

- Calculate the speed of a satellite 500 km above the earth's surface.

$$r = 500000\ m + 6.38 \times 10^6\ m = 6.88 \times 10^6\ m$$

$$G = 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$

$$M = 5.98 \times 10^{24}\ kg$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}\right)(5.98 \times 10^{24}\ kg)}{6.88 \times 10^6\ m}} = 7614\ m/s$$

5-03 Satellites

- Find the mass of a black hole where the matter orbiting it at $r = 2.0 \times 10^{20}$ m move at speed of 7,520,000 m/s.

$$v = \sqrt{\frac{GM}{r}}$$

$$7520000\ \frac{m}{s} = \sqrt{\frac{\left(6.67 \times \frac{10^{-1}\ Nm^2}{kg^2}\right)M}{2.0 \times 10^{20}\ m}}$$

$$5.655 \times 10^{13}\frac{m^2}{s^2} = \frac{\left(6.67 \times 10^{-11}\frac{Nm^2}{kg^2}\right)M}{2.0 \times 10^{20}\ m}$$

$$1.131 \times 10^{34}\frac{m^3}{s^2} = \left(6.67 \times 10^{-11}\frac{Nm^2}{kg^2}\right)M$$

$$M = 1.70 \times 10^{44}\ kg$$

5-03 Satellites

- Centripetal force is provided by gravity

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

- Cancelling the m's and a factor of r gives

$$v^2 = \frac{GM}{r}$$

- Speed is distance over time

$$v = \frac{2\pi r}{T}$$

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

- Re-arranging gives Kepler's Third Law

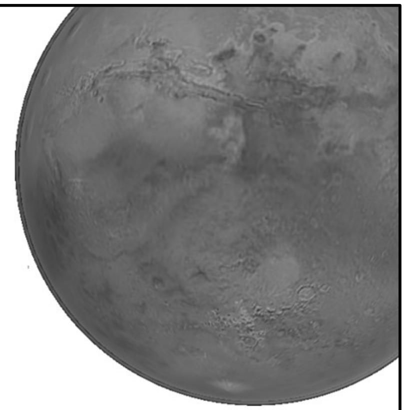
$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

- For circular orbits

$$T^2 = \frac{4\pi^2}{GM} r^3$$

T is period and r is average orbital radius

5-03 Satellites



- Use the data of Mars to find the mass of sun.
  Mars, $r = 2.279 \times 10^8$ km, $T = 1.881$ y

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$r = 2.279 \times 10^8\ km = 2.279 \times 10^{11}\ m, T = 1.881\ y = 59359845.6\ s$$

$$\frac{(59359845.6\ s)^2}{(2.279 \times 10^{11}\ m)^3} = \frac{4\pi^2}{\left(6.673 \times 10^{-11}\ \frac{Nm^2}{kg^2}\right) M}$$

$$235129.2 \frac{m^3}{kg}\ M = 4.6730 \times 10^{35}\ m^3$$

$$M = 1.99 \times 10^{30}\ kg$$

5-03 Satellites

- Since satellites are moving only under the influence of gravity, and the acceleration points towards earth, satellites are in freefall
- Astronauts in the space shuttles and international space station seem to float
- They appear weightless
- They are really falling
    - Acceleration is about g towards earth

5-03 Satellites

… they were finally able to close and repressurize the hatch. Several months later a new team of cosmonauts returned and found the hatch impossible to permanently repair. Instead they attached a set of clamps to secure it in place.

It is this set of clamps that Linenger and Tsibliyev are staring at uneasily seven years later. To his relief, the commander opens the hatch Without incident and crawls outside onto an adjoining ladder just after nine o'clock. Linenger begins to follow. Outside the Sun is rising. The Russians have planned the EVA at a sunrise so as to get the longest period of light. But because of that, Linenger's first view of space is straight into the blazing Sun. "The first view I got was just blinding rays coming at me," Linenger told his postflight debriefing session. "Even with my gold visor down, it was just blinding. [I] was basically unable to see for the first three or four minutes going out the hatch."

From *Dragonfly: NASA and the Crisis Aboard Mir* by Bryan Burrough

5-03 Satellites

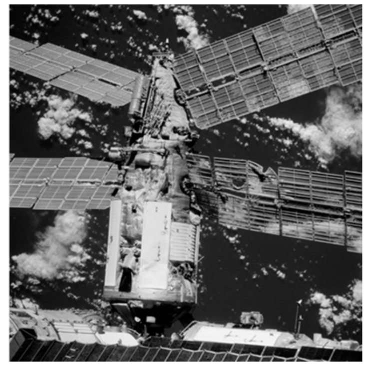
The situation only gets worse once his eyes clear. Exiting the airlock, Linenger climbs out onto a horizontal ladder that stretches out along the side of the module into the darkness. Glancing about, trying in vain to get his bearings, he is suddenly hit by an overwhelming sense that he is falling, as if from a cliff. Clamping his tethers onto the handrail, he fights back a wave of panic and tightens his grip on the ladder. But he still can't shake the feeling that he is plummeting through space at eighteen thousand miles an hour. His mind races.

You're okay. You're okay. You're not going to fall. The bottom is way far away.

From *Dragonfly: NASA and the Crisis Aboard Mir* by Bryan Burrough

5-03 Satellites

And now a second, even more intense feeling washes over him: He's not just plunging off a cliff. The entire cliff is crumbling away. "It wasn't just me falling, but everything was falling, which gave [me] even a more unsettling feeling," Linenger told his debriefers. "So, it was like you had to overcome forty years or whatever of life experiences that [you] don't let go when everything falls. It was a very strong, almost overwhelming sensation that you just had to control. And I was able to control it, and I was glad I was able to control it. But I could see where it could have put me over the edge."



From *Dragonfly: NASA and the Crisis Aboard Mir* by Bryan Burrough

5-03 Satellites

The disorientation is paralyzing. There is no up, no down, no side. There is only three-dimensional space. It is an entirely different sensation from spacewalking on the shuttle, where the astronauts are surrounded on three sides by a cargo bay. And it feels nothing—*nothing*—like the Star City pool. Linenger is an ant on the side of a falling apple, hurtling through space at eighteen thousand miles an hour, acutely aware what will happen if his Russian-made tethers break. As he clings to the thin railing, he tries not to think about the handrail on Kvant that came apart during a cosmonaut's spacewalk in the early days of Mir. Loose bolts, the Russians said.

Loose bolts.

From *Dragonfly: NASA and the Crisis Aboard Mir* by Bryan Burrough